Sum of Squares

Problem: Show that the sum of the first n integer square is

$$S_2 = 1^2 + 2^2 + 3^2 + \dots\dots\dots\dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Where, $S_0 = 1^0 + 2^0 + \dots\dots\dots\dots\dots + n^0 = n$ and

$$S_1 = 1^1 + 2^1 + \dots\dots\dots\dots\dots + n^1 = \frac{n(n+1)}{2}$$

Solution: *Proof by induction.*

Let, $Q(n) = 1^2 + 2^2 + 3^2 + \dots\dots\dots\dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

And

$Q(n+1) = 1^2 + 2^2 + \dots\dots\dots + n^2 + (n+1)^2 = \frac{(n+1)\left((n+1)+1\right)(2(n+1)+1)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$

Let's check, $Q(1) = \frac{1 \cdot 2 \cdot 3}{6} = 1$, which is true.

Now assume $Q(n)$ is true, let's prove $Q(n+1)$:

$$Q(n+1) = 1^2 + 2^2 + 3^2 + \dots\dots\dots\dots\dots + n^2 + (n+1)^2$$

$$= \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}$$

$$= \frac{(n+1)[n(2n+1) + 6(n+1)]}{6}$$

$$= \frac{(n+1)(2n^2 + n + 6n + 6)}{6}$$

$$= \frac{(n+1)(2n^2 + 7n + 6)}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

$$= \frac{(n+1)(n+2)[2(n+1)+1]}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

Hence proved by induction.